Application of the bending theory on square-hollow sections made from high-strength steel with a changing angle of the bending plane

M. Hudovernik, F. Kosel, D. Staupendahl, A.E. Tekkaya, K. Kuzman

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The bending behaviour of thin-walled profiles made from high-strength steels with respect to a changing angle of the bending plane or, in other words, the rotation of the section geometry on the longitudinal axis of the profile, has not yet been fully characterized. The investigations presented in this paper lead to an improvement of the description for bending square hollow sections under unified and constant loading conditions, and contribute to the general understanding of such bending problems. The methodological approach is based on analytic, numerical, and experimental analysis. The analytical formulation is primarily built on the principle of a simplified cantilever beam model. Bending curvatures are assumed to be generated with constant radii of curvatures. The change of the angle of the section, with respect to the direction of bending, is applied before bending and remains unchanged throughout the process. In this way, the effects of a changing angle with regard to the direction of bending are analyzed for several constant curvatures and angles of 2D bent profiles. With a clear understanding of the 2D bending of high-strength profiles, the same principles can also be used incrementally for analyses of 3D bending. The analytical theory is tested with an emphasis on using profiles with high-strength material properties compared to profiles made from standard low-carbon steel, by using the innovative torque superposed spatial (TSS) bending method. The results are supported by FE models generated with the Abaqus numerical FEM tool and verified with the results of actual experiments.

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1. Introduction

The forming behaviour of thin-walled square hollow profiles is limited by several technological aspects. During bending, their behaviour tends to relatively rapid distortions of the cross-section geometry and the occurrence of buckling, depending on the type of material, type of technology used and the size of the curvature applied. Hollow square profiles are preferably processed with the use of kinematic bending technologies based on two-, three-, or other multi-roll bending processes, such as the TSS. These allow a continuous displacement of the profile through a significantly smaller forming zone (compared to conventional technologies like press bending, stretch bending and NC bending of tubes), and smaller influence of localization of strains with a minimized effects of friction, which can also drastically influence the growth of geometrical distortions if process parameters are considered incorrectly. General response of profiles subjected to bending strongly depends on material properties and geometrical attributes, like the wall-thickness of the section of the profile. Profiles made from high-strength materials offer a great potential for the increase of the lightweight properties due to the possibility of minimizing the wall-thicknesses without any significant loss of the stiffness of the profile. On the other hand, such profiles represent a difficult task for any conventional bending technology. The importance of the use of kinematic bending methods in manufacturing of 2D or 3D bent contours of long profile components with closed hollow sections, therefore, grows even larger. In order to achieve complex spatial contours, the process for 3D bending of profiles requires an application of rotation of the section geometry, at the forming zone on the longitude axis of the profile, with regard to the bending plane. Effects of such rotations during bending of profiles have not yet been fully characterized and investigated. In order to fully understand effects of such angular changes on the response of material and the geometry of profiles during bending, it is best, to first analyze the bending behaviour during various conditions for continuous 2D bending, at different angles of the bending plane.
angle of the section’s geometry with respect to the bending plane. Consequently, the use of the TSS bending method permits the manufacturing of long profile and tube components with arbitrary 2D geometry, when the position of HTB is constant during the process, and 3D contours, when the position of HTB changes during the process.

Several published state of the art investigations deal with the analysis of bending of beams, tubes and profiles and offer a wide range of data concerning the modelling of various manufacturing technologies and general bending behaviour. Kosel et al. (2011) presents an analytical formulation based on the large displacement theory and elastic–plastic material with linear hardening properties for bending of beams at repeated loading and unloading conditions. Analytical solutions for the pure bending of hollow square sections, at a zero degree bending plane were presented by Megharbel et al. (2007). A similar process was presented by Quareshi (1999) for bending of circular sections with plastic hardening behaviour. A combined approach of using analytical and numerical analysis of bending of tubes and profiles were presented by Paulsen and Welo (1994). Li et al. (2010) described the behaviour of thin-walled tubes under push loading conditions with rotary draw bending. Zhan et al. (2006) and Jiang et al. (2010) presented similar studies concerning the prediction of springback behaviour in different processes for the bending of circular tubes. Chatti et al. (2010), Hermes (2011) and Staupendahl et al. (2011) were among the first to analyze the capabilities and forming limits of the TSS bending method by using analytical and experimental methodologies. The effects of elasticity of tools and profiles on the final 2D or 3D profile contours, together with the effect of principle concerning superposition of stresses on springback, were analyzed for the bending of profiles from standard low strength steel. Due to the strong dependency of the final geometrical outcome on elasticity of tools and the material of the profile, semi-analytical models with corresponding correction factors are introduced in Chatti et al. (2010). These are successfully applied to the process of generating NC codes needed to control movements of the machine, and for prediction of final geometrical outputs.

The behaviour of profiles with square hollow sections may significantly differ when bending is applied on profiles from different types of steel at different angles of the bending plane. Such arbitrary bending conditions are achieved and tested with the use of the TSS bending method, previously not yet fully investigated. In this paper, the problem of bending profiles is first formulated by using the
idealized cantilever beam model shown in Fig. 2, where distances between all FR pairs are equal to \( l_1 = 250 \text{ mm}, l_2 = 325 \text{ mm} \) and \( l_3 = 395 \text{ mm} \). Outer dimensions of all FRs are \( D_o = 200 \text{ mm} \), smaller rolls mounted inside the BH are \( d_o = 68 \text{ mm} \). After validation with experimental results the same formulation is adapted by using stiffness correction factors shown in Chatti et al. (2010) to achieve an accurate model capable of analysing bending behaviour of profiles from high strength steels. Numerical software tool Abaqus is also used for the purpose of providing more detailed analysis of the response of square hollow sections to various bending conditions based on the TSS bending principles.

2. Formulation of the problem

By using the TSS bending method shown in Fig. 1, the radius of curvature of bent profile continuously decreases with the increase of BH displacement on \( w \)-axis. The relationship between BH displacement and the radius is shown in Fig. 2. In order to provide constant bending conditions the BH must first travel on \( w \)-axis from the initial position, when the profile is straight, to the final position \( w_f \). Elastic deformations of tools are hereby neglected, and the forming zone is assumingly positioned directly in-between the last pair of feed rolls or FR Pair 3. A certain length of the profile (i.e. equal to the distance \( l_3 \) between BH and last pair of feed rolls) must be displaced to achieve constant bending conditions: bending moment \( M_b \) and radius \( R \), at that specific zone. Constancy of radius is provided for as long as the BH remains on the same point until any further displacement ensures the change of its position. For the TSS technology, the angle of the bending plane is numerically controlled by the torsion-bearing element. It provides a constant or variable angle, resulting in either 2D or 3D bent profile geometries. A rough estimate of theoretical loaded radius \( R_l \) of curvature and angle of bending are defined with Eqs. (1) and (2) previously presented by Hermes (2011).

\[
R_l = \frac{w_f}{2} + \frac{l_3^2}{2w_f} 
\]

\[
\theta = \arccos \left( \frac{l_3^2 - w_f^2}{l_3^2 + w_f^2} \right) 
\]

The formulation for bending of thin-walled square hollow profiles is built according to the following assumptions:

- Theory of large displacements and small strains is applied.
- Section does not deform during and after bending.
- Bending radius \( R \) and moment \( M_b \) in the forming zone are considered to be constant.
- Profile is initially considered as straight.
- Material is considered as isotropic, homogeneous, with smooth strain hardening properties.
- Elasticity is defined by Hook’s law described in Eq. (3).
- Plasticity is defined by the power law hardening rule described in Eq. (4).

\[
\sigma_{el} = \varepsilon \varepsilon, \quad \text{for } \varepsilon \leq \varepsilon_y 
\]

\[
\sigma_y = C\varepsilon^n \quad \text{for } \varepsilon \geq \varepsilon_y 
\]

Elastic and yield stress are denoted by \( \sigma_{el} \) and \( \sigma_y \), Young’s modulus with \( E \), further, \( C \) denotes material constant, \( \varepsilon \) strains, \( \varepsilon_y \) the yield strain at \( \sigma_y \), and \( n \) is the hardening exponent. Axial strains are defined from conventional bending theory with Eq. (5).

\[
\varepsilon = \frac{x}{R} 
\]

In analytical formulation, the bending plane angle \( \alpha \) is considered to be constant during bending; however, the same formulation can also be applied incrementally for values ranging between 0° and 45° when \( \alpha \) changes during the process. All inner and outer transverse and axial forces are considered to be in equilibrium. Moreover, the constancy of loaded radius \( R \) and the bending moment \( M_b \) are considered in the core of the forming zone with every increment of displacement of the BH. Elastic and plastic portions of the total bending moment are defined in Eqs. (6)–(13), according to the given angle of bending plane or cross-section position as shown in Fig. 3a–c.

The bending moment \( M_b \) acts in \( y \)-axis of the cross-section of the profile as a result of inner axial force \( N \). The total \( M_b \) is combined from elastic and plastic components as shown in Eq. (7).

\[
dM_b = \int^N x \cdot dN 
\]

\[
M_b = M_{el} + M_y 
\]
General formulation for the plane bending of square section at 0° angle, as shown in Fig. 3a is presented in Eqs. (8) and (9) (Megharbel et al., 2007). Elastic moment \( M_b \) acts in the region from the centre of gravity of the section geometry to the \( x_e \). Plastic moment \( M_p \) acts in two regions: first from \( x_e \) to \((a-t)\) and second, from \((a-t)\) to \( a \), where \( a \) denotes one half of the outer dimension of a square, and \( t \) is the wall-thickness of the hollow geometry.

\[
M_b = 2 \int_0^{x_e} E \left( \frac{x}{R} \right) 2tx \, dx + 2 \int_{x_e}^{(a-t)} E \left( \frac{x}{R} \right) n \left[ \frac{2tx}{\cos \alpha} \right] dx + 2 \int_{(a-t)}^{a} E \left( \frac{x}{R} \right) n \left[ \frac{2tx}{\cos \alpha} \right] dx
\]

(8)

With the input of Eqs. (3)–(5) in Eq. (8), together with determination of \( dA \) for the given section position shown in Fig. 3a, Eq. (9) is formed.

\[
M_b = 2 \int_0^{x_e} E \left( \frac{x}{R} \right) 2tx \, dx + 2 \int_{x_e}^{(a-t)} E \left( \frac{x}{R} \right) n \left[ \frac{2tx}{\cos \alpha} \right] dx
\]

(9)

The \( x_e \) defines the elastic limit of the section or the point where \( M_d = M_b \) and \( \sigma_{\alpha} = \sigma_y \). It depends on material properties, the size of radius of curvature \( R \), and the angle of the bending plane \( \alpha \). The relationship between elastic and plastic fields at the point \( x_e \) is shown in Eq. (10) gained by combining conditions of Eqs. (3), (4) and insertion of Eq. (5) additionally calibrated due to the influence of angle \( \alpha \).

\[
x_e \alpha^{-1} = \frac{E}{C} \frac{R_{\alpha}^{n-1}}{\alpha(1-\alpha)}
\]

(10)

Changing the angle of the cross-section geometry affects the output response of the bending process and the change of geometrical boundaries. Fig. 3a and c shows the initial and final rotated sections with regard to the bending plane, whereas Fig. 3b describes conditions occurring between the two extremes. In the latter case, the bending moment \( M_b \) is defined according to the geometrical regions or boundaries of the section denoted by \( x_i \) to \( x_m \), given with the following relations:

\[
- x_i = (a-t) \sqrt{2} \cdot \cos(45^\circ + \alpha),
- x_{II} = (a-t) \sqrt{2} \cdot \cos(45^\circ - \alpha),
- x_{III} = a \sqrt{2} \cdot \cos(45^\circ - \alpha)
\]

Depending on the angle \( \alpha \), as shown in Fig. 3b for \( 0 \leq \alpha \leq 45^\circ \) and \( x_e \leq x_i \), a general form of bending moment \( M_b \) is defined with Eq. (11).

\[
M_{b;45deg} = 2 \int_0^{x_e} E \left( \frac{x}{R} \right) 2tx \, dx + 2 \int_{x_e}^{(a-t)} E \left( \frac{x}{R} \right) n \left[ \frac{2tx}{\cos \alpha} \right] dx
\]

(11)

The same condition can be applied to cases where \( x_e \leq x_i \), with the exception that \( x_i \) is then simultaneously included in the definition of elastic component of the bending moment and is, therefore, neglected. For the case where \( \alpha = 45^\circ \) and \( x_i = 0 \), the \( M_b \) is defined with Eq. (12).

\[
M_{b;45deg} = 2 \int_0^{x_e} E \left( \frac{x}{R} \right) \left( \frac{1}{\cos \alpha} + \frac{1}{\sin \alpha} \right) \, dx
\]

(12)

Since \( \alpha = 45^\circ \), the expression \((1/\cos \alpha + 1/\sin \alpha)\) equals \( 2 \sqrt{2} \). Eq. (12) is then written in a simplified form of Eq. (13).

\[
M_{b;45deg} = 4 \sqrt{2} \int_0^{x_e} E \left( \frac{x}{R} \right) \, dx + 4 \sqrt{2} \int_{x_e}^{(a-t)} E \left( \frac{x}{R} \right) n \left[ \frac{2tx}{\cos \alpha} \right] dx
\]

(13)

The TSS method can be presented by a simplified cantilever beam if the system for providing the feed and the bending of the profile is considered as ideally rigid, as shown in Fig. 2. Due to the adaptive behaviour of the BH, the total bending force \( F_b \) consists of two components: first, the \( F_{bw} \) acting transversely to feed of the profile and, second, the eccentric axial force \( F_{bc} \) acting in the opposite direction of feed of the profile. During a state of equilibrium between internal and external loads, two main components of bending moment \( M_b \), Eq. (14) are consequently generated: first, the bending moment portion \( M_{bw} \) as a result of \( F_{bw} \) and, second, the \( M_{bc} \) due to the effect of \( F_{bc} \). With the introduction of polar coordinates,
the $M_b$ is also defined with Eq. (15).

$$M_b = M_{bw} + M_{bc} = F_{bw} \cdot l_3 + F_{bc} \cdot w_f = F_b \cdot \cos \theta \cdot l_3 + F_b \cdot \sin \theta \cdot w_f$$

(14)

$$M_b(\varphi) = F_b \cdot R_l \cdot \sin(\theta - \varphi)$$

(15)

In the domain of idealized cantilever beam model, the maximum $M_b$ is theoretically induced in the centre of forming zone, directly between the last pair of feed rolls when bending $\theta = 0$. The $M_b$ then gradually decreases with increasing bending angle, and reaches zero at the point where distance between the BH and feed rolls is zero. Due to constant displacement of the profile along the longitudinal axis, the entire curvature length of the profile is assumed to be subjected to the same bending moment to generate constant radius. By combining Eqs. (9), (11), (13) and (14) or (15), a general formulation is obtained to determine the bending force $F_b$, Eq. (16).

$$F_b = \frac{M_{b_\text{codeg}}}{l_3 \cdot \cos \theta + w \cdot \sin \theta} = \frac{M_{b_\text{codeg}}}{R_l \cdot \sin \theta}$$

(16)

Bent profiles are subjected to springback generated by the remaining $M_d$ after all constraints are removed. In terms of $R_d$, the unloaded geometry of the profile can be obtained from conventional bending theory with a relationship in Eq. (17), where $\Delta \kappa$ denotes the difference of loaded and unloaded curvatures.

$$\Delta \kappa = \frac{1}{R_l} - \frac{1}{R_d} = \frac{M_b}{EI}$$

(17)

Stiffness correction factors are introduced for the TSS bending process with Eqs. (18) and (19) (Chatti et al., 2010). The $\Delta \xi_t$ concerns the influence of elasticity of tools on bending and denotes stiffness correction factor of tools. The $\Delta \xi_p$ concerns influence of elasticity of bent material and denotes the stiffness correction factor of the profile (Chatti et al., 2010). To integrate these factors into the formulation, Eqs. (1) and (2) are first used to define $M_b$, after which the $F_b$ is inserted into Eqs. (16) and (19).

$$\Delta \xi_t = 1496 \times 10^{-3} \cdot F_b^2 + 3.674 \times 10^{-4} \cdot F_b$$

(18)

$$\Delta \xi_p = \frac{F_b}{3 \cdot EI} \left( l_1(l_2 + h)^2 + (l_2 + h)^2 \left( \frac{l_1(l_2 + h)^2 + (3/2)l_2(l_2 + h)^2}{l_1 + l_2} \right) \right)$$

(19)

In order to successfully analyze the effect of stiffness of the profile in the cantilever beam model, Eq. (19) is adjusted by extracting the cantilever beam stiffness portion, Eq. (20), denoted by $\Delta \xi_{cb}$, already included in the formulation of the idealized model. Final corrected stiffness factor of the profile $\Delta \xi_{corr}$ is presented in Eq. (21), which is then inserted into Eq. (22) to achieve the final corrected loaded radius (Chatti et al., 2010).

$$\Delta \xi_{cb} = \frac{F_b \cdot l_3}{3 \cdot EI}$$

(20)

$$\Delta \xi_{corr} = \Delta \xi_p - \Delta \xi_{cb}$$

(21)

$$R_{corr} = \frac{F_b^2}{2(w_f - \Delta \xi_t - \Delta \xi_{corr})} + \frac{l_3^2}{2}$$

(22)

3. Materials and methods

The analytical models presented here are supported with the use of the Abaqus FEM computational tool and validation by a comparison to results of experimental analysis. Experimental work was performed by bending of seamless and welded square hollow profiles with 40 mm × 40 mm outer dimensions, manufactured from two types of steel and two different types of manufacturing processes. Seamless profiles with a 2.5 mm wall thickness are manufactured from high-strength steel MW700L Z3 – named specifically by the manufacturer Salzgitter, which is by properties similar to the steel S700MC (Norm: DIN EN 10149-2). Welded profiles are manufactured from a 3 mm thick sheet from standard construction low-carbon steel S235JR (Norm: EN 10025:2004), with a combination of two processes: roll-forming and welding. General material properties, presented in Table 1, are extracted from standard tensile tests shown in Fig. 4. Flow curves for both types of steel, also shown in Fig. 4, are extrapolated by using the power law conditions, Eq. (4), which can be successfully applied to materials with presumably good homogeneous and isotropic structure and smooth hardening behaviour. During actual bending of such profiles with the use of the kinematic TSS bending method, mainly small strain conditions occur. Therefore, flow curves for both types of steel shown in Fig. 4 are presented only for smaller values of true strains. A dimensionless factor representing relative wall-thickness of the profile, Eq. (23), is introduced for the purpose of investigation to describe the geometrical properties of individual sections of thin-walled profiles.

$$\lambda = \frac{2a}{l}$$

(23)

3.1. Experimental analysis

Experimental analysis is performed with the use of TSS bending method technology as described by Staupendahl et al. (2011). NC software programmes based on the pre-defined CAD geometry are generated to provide optimal bending conditions to achieve geometries with different constant radiiues of curvature. This is necessary to assure accurate control of constant feed of the profile, movements of BH along w-axis and constant or variable position of

![Fig. 4. Stress strain relations from tensile tests of MW700L Z3 and S235JR steel.](Image)
the torsion bearing which provides accurate angle of the bending plane $\alpha$. The NC code is generated with a digital mock-up system of CATIA V5 (Dessault Systems) kinematics module, as shown in Fig. 5, with a continuous displacement of pre-determined curved CAD geometry through the mock-up setup of virtual sensors at constant velocity. This assures the repeatability of bending of profiles in two phases: first by bending from initially straight profile to the final position of BH at the distance $w_f$ and, second, by bending at the fixed final position at continuous feed of the profile to achieve curvature with constant radius. Experimental analysis is performed according to the factorial design shown in Table 2, where material type, BH displacement $w_f$, and bending plane angle $\alpha$ represent process factors at two levels denoted with numerical values. The feed of the profile is set at constant velocity of 10 mm/s. Bending is set according to the NC programme for two BH displacement levels ($w_f = 35$ mm, and $w_f = 200$ mm) and at two levels of bending plane ($\alpha = 0^\circ$ and $\alpha = 45^\circ$). Three repeated experimental runs are performed for each of the parameter settings. In addition to the experimental work based on the factorial design, shown in Table 2, two extra sets of experimental runs are performed for bending at $w_f = 65$ mm and $w_f = 118$ mm, at $\alpha = 0^\circ$ and $\alpha = 45^\circ$. These results contribute to a more detailed understanding of the response of square hollow profiles during bending and are further used for validation of numerical results.

The bending force $F_{bend}$ is measured directly on the $w$-axis during the process by using the Type 9041A Kistler piezo-load washer mounted on the axis of displacement unit. Constant conditions during experimental bending are assured to provide curvatures of lengths equal to at least the distance between $w_f$ and FR pair 3. Bent and unloaded geometries, in terms of $R_{UL}$, are analyzed with using a digital GOM Atos measuring system after the bending process is finished and profiles are free of any constraint of the TSS machine. Experimental unloaded radiiues, or $R_{UL}$’s, are determined accordingly to the position of the neutral axis of each bent profile.

Table 2

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$^a$ Material (1) = M8700L Z3, Material (2) = S235JR, $w_f(1) = 35$ mm, $w_f(1) = 200$ mm, $\alpha(1) = 0^\circ$, $\alpha(1) = 45^\circ$.

3.2. Numerical models

Numerical analysis is performed with the use of Abaqus software. The von Mises yield criteria and isotropic hardening rule are assigned to material models. All tools, such as the rolls of the BH and feed rolls or FR, are defined with analytically rigid properties. Due to the nature of quasi-static bending process and the availability of a decreasing number of elements without a significant risk of decreasing computational accuracy, the implicit code of Abaqus software is used. Quadratic doubly curved thick shell elements based on Mindlin–Reissner theory are used for discretization of the deformable profile. The section is defined with Gauss integration rule and five integration points through the thickness of the shell. A surface-to-surface contact method based on penalty formulation is applied to define contacts between all rolls and the profile. Hard contact properties with penalty pressure over-closure and linear constraint enforcement are assigned to define contact behaviour in the normal direction and penalty friction formulation with Coulomb friction model to define contacts in the tangential direction. Rotational degrees of freedom for symmetry axes of all the analytical rigid rolls are unconstrained. This allows their rotation, consistent to constant movement of the profile during the contact. Such contact conditions assure the development of contact forces that contribute to higher quality of resulted response. The feed of the profile is simulated by assigning constant velocity via the definition of the boundary condition in the direction of $c$-axis, directly on the transverse edge at the furthest end from the forming zone of the profile. The BH element is based on properties of an actual mechanical part, assembled from four individual part instances (rolls, inter-connected by multi-point constraints and connector assignments). Individual BH rolls are linked to one single reference point BHref placed in the centre of the BH assembly, as shown in Fig. 6. This reference point is then used for control of BH movements along bending $w$-axis. The FEM analysis of bending of square hollow profiles is performed by using the same parameter settings as those for experimental analysis, with additional level of the angle of bending plane, at $\alpha = 22.5^\circ$.

4. Analysis and results

All results of the analytical and numerical models are verified by a comparison to experimental results, collected with the methods.
described in Section 3.1. Two criteria for validation are used. First, the unloaded geometry $R_{UL}$, which is analyzed with the use of a GOM Atos system, and second, the output bending force $F_{bw}$ – measured directly on the $w$-axis throughout the actual process, together with the angle of adaptive rotation of BH around its vertical axis. Experimental bending results, in terms of constant unloaded radius $R_{UL}$, bending force and momentum, are analyzed for the parameter settings for bending at $w_f = 35$ mm, $w_f = 65$ mm, $w_f = 118$ mm, and $w_f = 200$ mm, at angles $\alpha = 0^\circ$ and $\alpha = 45^\circ$, as described in Section 3.1. In contrast, results of analyses gained by FEM and analytical models are also presented for an additional angle of the bending plane at $\alpha = 22.5^\circ$.

4.1. Results for geometry and force output

The experimental unloaded geometry tends to behave similarly for both materials at $\alpha = 0^\circ$ and $\alpha = 45^\circ$, as shown in Figs. 7 and 8. By comparison, small BH displacements show higher differences between experimental results for bending MW700L Z3, than results for S235JR. The output $R_{UL}$ for MW700L Z3 profiles at $w_f = 35$ mm and $\alpha = 0^\circ$ is, on average, 14% higher than $R_{UL}$ from bending at $\alpha = 45^\circ$ – Fig. 7. The difference between the two is decreased with an increase of $w_f$ and reaches less than 1% at the maximum displacement of $w_f = 200$ mm. The differences in $R_{UL}$ response for bending S235JR profiles, using same conditions, are only 1.8% at $w_f = 35$ mm and reach even smaller values at $w_f = 200$ mm as shown in Fig. 8. The numerical unloaded geometry at $w_f = 35$ mm and $\alpha = 0^\circ$ reaches states with approx. 10% lower $R_{UL}$ than the geometry at $\alpha = 45^\circ$, whereas, in reality, this behaviour is reversed with approximately the same values.

The difference in numerical results at all three levels of $\alpha$ show practical irrelevance when $w_f \geq 118$ mm for MW700L Z3 and S235JR profiles. In comparison to experimental values of $R_{UL}$’s, FEM results are in general slightly smaller than experimental ones, with a maximum of 20% at $w_f = 35$ mm in the case of MW700L Z3, and a maximum of 19% in the case of S235JR. A comparison of analytical results shown in Figs. 7 and 8, achieved with the use of Eqs. (1) and (17) based on an idealized rigid cantilever beam, reveals a consistent behaviour with minor differences in terms of $R_{UL}$ at all three levels of $\alpha$. The proximity of these analytic results is closer to FEM than the experimental results in all cases. When analytical values of $R_{UL}$ are compared to experimental ones, the average difference between results of bending MW700L Z3 profiles at $\alpha = 0^\circ$ and $\alpha = 45^\circ$ is 23% for $w_f = 35$ mm, and 3% for $w_f = 200$ mm. For bending S235JR profiles in the same conditions, a 17% difference occurs at $35$ mm BH displacement and 8% difference at $200$ mm BH displacement. The largest difference in geometry occurs at minimal BH displacement. In contrast, the closest proximity with minimal deviations is reached at maximum displacement.

Results for the validation criterion $F_{bw}$ are shown in Fig. 9. Experimental and numerical values of $F_b$ and $M_b$, shown in Figs. 10 and 11, strongly depend on the accuracy of the $F_{bw}$ measurement and calculations of the angle $\theta$. They are derived with the use of the analytical formulation of the cantilever beam model shown in Fig. 2. For bending of high strengths steel MW700L Z3 profiles, analytic results
4.2. Analysis of results

Elastic deformations and deflections of profiles during bending at small displacements have a substantial impact on output responses for both validation criteria. This effect is reduced after a certain length of the profile undergoes forming and at increasing displacements. It is minimized at maximum \( w_f \). The formulation for the prediction of profile bending, as analyzed for the use of the TSS bending method and presented in this paper, is suitable for applications of larger BH displacements. All experimental results, especially when \( w_f \leq 65 \text{ mm} \), show significant differences of force and geometrical results compared to analytical solutions. For the purpose of improving the analytical prediction of the geometry, adjustments that concern the stiffness properties of the material of the profile and the tools are introduced in the process as previously described by Chatti et al. (2010) in Eqs. (18)–(22). Figs. 12 and 13 show comparisons of the initial and improved analytical solutions for the prediction of the unloaded geometry of the bent profiles. At \( w_f = 35 \text{ mm} \), the initial difference between experimental and analytical values of \( R_{ul} \) for MW700L Z3 profiles reaches up to 23% for bending tests at \( \alpha = 0^\circ \), and 20.4% at \( \alpha = 45^\circ \). For the bending S235JR profiles, a deviation of 17% occurs at \( \alpha = 0^\circ \), and a deviation of 15% at \( \alpha = 45^\circ \). By applying stiffness correction factors, these differences in output geometry are drastically reduced down to 0.5% at both \( \alpha = 0^\circ \) and \( 45^\circ \) for MW700L Z3 profiles, and down to 7.5% for \( \alpha = 0^\circ \) and 10.5% for \( \alpha = 45^\circ \) for S235JR profiles.

As shown in Figs. 9–13, bigger deviations between analytic and experimental bending force and bending moment results occur for bending profiles made from S235JR than from MW700L Z3. This may be due to different reasons. One can be attributed to the effects created by the welding seam, placed on one of the surface sides of the actual S235JR profiles, leading to a non-uniform bending behaviour. In contrast, the MW700L Z3 profiles are cold drawn out of seamless material. The analytical and numerical models neglect this particular detail. A second reason for an increase of deviations may be related to the fact that the power law hardening rule is insufficient for material modelling of soft state steels like S235JR. Other material models should, therefore, also be examined in future.
investigations. A third reason concerns the wall-thickness properties of profiles. The thickness of S235JR profiles is slightly greater than the thickness of MW700L Z3 high strength steel profiles. The influence of shear effects on the final outcome, which is neglected in the formulation, may have a slightly larger impact on the final outcome than initially expected. In the FEM domain, transverse shear stiffness is included in the formulation through the use of discrete shell elements based on the Mindlin–Reissner theory. Consequently, all FE results, when compared to analytical results, show behaviour with a closer proximity to experimental values. In future analyses, significant emphasis should be placed on investigating effects of shear on bending of profiles with different wall-thickness properties. Furthermore, due to the complex state of stresses over the cross-sections of square hollow profiles, a considerable uncertainty concerning behaviour of geometry of the cross-section of the profile during bending also arises. The effects of bending parameters on cross-section deformation should also be closely examined in the future.

5. Conclusion

The analysis of bending of square hollow, thin-walled profiles made from high-strength steel, at different constant curvatures and different angles of the bending plane ($\alpha = 0^\circ$, $\alpha = 22.5^\circ$ and $\alpha = 45^\circ$) in comparison to S235JR steel is presented in this paper. One analytical formulation, based on the cantilever beam model (Fig. 2), is tested in two variations. One concerns the idealized rigid properties of tools, while the other includes the stiffness properties of tools and material of the profile material as investigated by Chatti et al. (2010) for the TSS bending method. The latter improved the results of the initial model since analytical results concerning force response show a much closer proximity to experimental values. Minimal deviations occur for the high-strength profiles. The deviations are larger during the bending of thicker-walled S235JR profiles. This implies the possibility of an increasing presence of shear effects. The investigation is also supported by the use of the Abaqus FE numerical computational tool. All FE models are based on the use of thick shell technology and include transverse shear stiffness properties. FE simulations are performed by using the same conditions as used for the experimental analysis, with one additional level of bending plane angle: $\alpha = 22.5^\circ$. Results concerning both force and geometrical response show a better proximity to the experimental results than the analytical results gained by using the initial form of a cantilever beam mode I. Such FE models can be used for further optimization and a detailed analysis of the state of stresses and strains over the cross-section at various combinations of process parameters, where $0^\circ < \alpha < 45^\circ$ and $0 < w_f < 200$ mm, in either 2D or 3D bending. Bending behaviour of thin-walled profiles with square hollow sections, as presented in this paper, has not yet been fully characterized for the use of kinematic TSS bending process. Therefore, all results shown hereby represent an important contribution to the overall characterization and understanding of such bending problems. The methodology and investigations presented hereby are to be further used for the analysis of the response of square hollow sections for free-form 3D bending of profiles during continuously changing conditions.

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